Solving each factor,

$$x=(\sqrt{3}\pm 2)y\ldots\ldots\ldots(4),$$
$$x=(-\sqrt{3}\pm 2)y\ldots\ldots\ldots(5).$$

From the simultaneous equations (2), (4) and (2), (5),

$$x=\frac{m}{6}(3\pm\sqrt{3}),\quad y=\frac{m}{6}(3\mp\sqrt{3}),$$

$$x=\frac{m}{2}(1\pm\sqrt{3}),\quad y=\frac{m}{2}(1\mp\sqrt{3}).$$

II. **Solution by S. A. COREY, Hiteman, Iowa.**

Adding $2x^2y^2$ to each member of (1),

$$x^4+2x^2y^2+y^4=16x^2y^2\ldots\ldots\ldots(3),$$

whence $x^2+y^2=\pm 4xy\ldots\ldots\ldots(4)$.

Squaring (2), we have

$$x^2+2xy+y^2=m^2\ldots\ldots\ldots(5);$$

substituting from (4), $6xy=m^2$, or $-2xy=m^2$,

whence, $x=\dfrac{m^2}{6y}$ or $-\dfrac{m^2}{2y}$.

By substituting these values of x in (2), we find without difficulty,

$$x=m(\tfrac{1}{2}\pm\sqrt{\tfrac{1}{12}}),\text{ or } m(\tfrac{1}{2}\pm\sqrt{3/4});$$
$$y=m(\tfrac{1}{2}\mp\sqrt{\tfrac{1}{12}}),\text{ or } m(\tfrac{1}{2}\mp\sqrt{3/4}).$$

Also solved by R. L. Borger, G. W. Greenwood, E. L. Sherwood, L. E. Newcomb, J. F. Lawrence, S. E. Harwood, Elmer Schuyler. G. B. M. Zerr, and J. Scheffer.

209. **Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, O.**

Prove that $(a^4+b^4+c^4+d^4)>4abcd$.

Solution by M. E. GRABER, A. M., Heidelberg University, Tiffin, O.

$(a^2+b^2)>2ab$ and $(c^2+d^2)>2cd$ from which, by multiplication, $(a^2c^2+b^2c^2+a^2d^2+b^2d^2)>4abcd\ldots\ldots\ldots(1)$.

Again $(a^4+c^4)>2a^2c^2$; $(b^4+c^4)>2b^2c^2$; $(a^4+d^4)>2a^2d^2$, and $(b^4+d^4)>2b^2d^2$.

By addition, $(a^4+b^4+c^4+d^4)>(a^2c^2+b^2c^2+a^2d^2+b^2d^2)$ and this in connection with (1) gives $(a^4+b^4+c^4+d^4)>4abcd$.

Also solved by G. W. Greenwood, R. L. Borger, E. L. Sherwood, L. E. Newcomb, Elmer Schuyler, J. H. Meyer, A. J. Haun, O. A. Laisant, J. Scheffer.

*** Dr. G. B. M. Zerr and Mr. J. F. Lawrence prove in general that $a_1+a_2+\dots\dots+a_n>n\surd(a_1a_2\dots\dots a_n)$. Dr. Zerr also proved that $(a_1{}^m+\dots\dots+a_n{}^m)/n>[(a_1+\dots\dots+a_n)/n]^m$.

210. Proposed by **W. J. GREENSTREET, A. M., Editor of The Mathematical Gazette, Stroud, England.**

The sum of five quantities and the sum of their cubes are both zero. Show that the sum of their fifth powers is a factor of the sum of any odd powers of the quantities.

Solution by **G. W. GREENWOOD, M. A. (Oxon), Professor of Mathematics and Astronomy in McKendree College, Lebanon, Ill.**

Denote the quantities by α, β, γ, δ, ε, and let the equation of which they are the roots be

$$x^5+ax^4+bx^3+cx^2+dx+e=0.$$

Then $$\Sigma\alpha=-a=0.$$

$$\Sigma\alpha^3=(\Sigma\alpha^2-\Sigma\alpha\beta)\Sigma\alpha-\Sigma\alpha\beta\gamma)=c=0.$$

Substituting the roots in turn for x, and adding, we get,

$$\Sigma\alpha^5+5e=0.$$

Multiply the equation by x^2, make the same substitutions, and we get in a similar manner,

$$\Sigma\alpha^7+b\Sigma\alpha^5+5e=0,$$

$$i.\ e.,\ \Sigma\alpha^7+(b-1)\Sigma\alpha^5=0.$$

Hence $\Sigma\alpha^5$ is a factor of $\Sigma\alpha^7$ and the process may be repeated indefinitely.

Also solved by Elmer Schuyler, G. B. M. Zerr, J. Scheffer.

GEOMETRY.

228 Proposed by **O.E. GLENN, A.M., Fellow in Mathematics, University of Pennsylvania, Philadelphia, Pa**

Given a point O without a circle S; two arbitrary lines through O cut S in the points A, A', and B, B', respectively. Prove, by pure geometry, that the four circles through OAR, OBR, $OA'R'$, $OB'R'$, respectively, intersect in points collinear with O; R and R' being points upon S arbitrarily chosen.

Solution by **T. L. CROYES, Paris, France.**

Let us take the inverse of the system with regard to O, and let the inverses of the five circles S, OAR, OBR, $OA'R'$, $OB'R'$, be a circle s, and the four right lines ar, br, $a'r'$, $b'r'$ (a, a', b, b', r, r' being the inverses of A, A', B, B', R, R').

By Pascal's theorem, the points of intersection $(ar,\ b'r')(br,\ a'r')$ are collinear with the point O.

237. Proposed by **S. A. COREY, Hiteman, Iowa.**

Let AB, BC, CD, DE, EA be the sides of a pentagon, plain or gauche. Double the length of CB and DE by extending from B and E to G and H, re-